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Finite Element Modeling of Reinforced Concrete Beams Externally Strengthened in Flexure with Side-bonded FRP Laminates

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ABSTRACT

Reinforced concrete (RC) beams are often strengthened through externally bonded (EB) fiber-reinforced polymer (FRP) systems. In such applications, improving the bending capacity of RC beams is of main interest and which is achieved via bonding EB-FRP systems to the soffit of beams. Access to soffit of beams can be restricted by existing structural members or limited (in case a RC beam span over neighboring compartments). In those scenarios, RC beams can still potentially be strengthened in flexure through longitudinal bonding of EB-FRP systems to the sides of beam's web. This paper examines critical parameters that influence the effectiveness of side bonded EB-FRP systems through a newly developed finite element (FE) model. This model utilizes state-of-art simulation techniques and is capable to trace the flexural behavior of RC beams externally strengthened with side-boned FRP laminates throughout all loading stages till failure. The model was validated by comparing with experimental data and the predicted and measured results were in good agreement. The validated model was then utilized to study the effect of concrete compressive strength, FRP material type, FRP size, and steel reinforcement ratio on the behavior of strengthened RC beams. Outcomes of this numerical investigation show the effectiveness of side-bonding FRP systems as an alternative to conventional soffit strengthening systems to improve the flexural capacity of RC beams.

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Keywords: reinforced concrete; flexural strengthening; side-bonded; fiber-reinforced polymers; finite element modeling.

1.0 Introduction

Most existing infrastructure in developed countries was put into service post World War II. As a result of increasing demands arising from ever booming population, when combined with natural deterioration (i.e. weathering, corrosion etc.) of construction materials, the level of service in civil construction continues to decline [1]. Hence, developing strategies and practical solutions to retrofit structures has been an emerging research area in the last three decades [2-8]. Extending service life of infrastructure can be achieved via two strategies, regular upkeep (maintenance), and retrofitting of damaged and aged structural components. Upkeep of infrastructure is often associated with increasing direct costs (attributed to yearly inflation) as well as indirect costs such as those arising from close-of-business or detouring (in case an infrastructure is required to be shut down in order for maintenance crews to carry out repairs). On the other hand, retrofitting of infrastructure provides a more suitable solution in which damaged structural systems can be upgraded for desired (i.e. future) level of service with minimum downtime and relatively lower overall costs, as compared to regular upkeep [9-11].

The recent advancement in material sciences has led to development of composites, most notably fiber-reinforced polymer (FRP) sheets and plates. Such FRP composite materials comprise of high strength continuous fibers (with load carrying abilities) embedded in a polymer matrix (resin). This composite material has superior mechanical properties, including high stiffness/strength to weight ratio, good tolerance to creep and fatigue, as well as resistance to

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corrosion and harsh environments [12]. All of above characteristics seem to suit strengthening applications and as such, FRPs present a unique opportunity to the retrofitting industry [13-16]. FRP strengthening systems (i.e. sheets, pates, bars etc.) are often attached to the exterior faces of RC structural members (i.e. beams, columns) or components (walls, frames) by means of special adhesives. Bonding such systems is quick and easy, and hence the use of FRP strengthening systems offer a quick solution to aging structures or those affected by trauma (i.e. earthquake) [17-20]. In fact, a close examination of open literature shows the sheer magnitude of work in this area, especially to successful retrofitting of severely damaged RC structures such as those subjected to extreme loading events such as fire and blast [21-24].

From the point of view of this study, retrofitting of horizontal (lateral) structural members (i.e. beams) is mainly associated with improving their bending and/or shear capacity. Since FRP sheets are usually unidirectional with fibers running in the longitudinal direction, then the EB-FRP systems are bonded (or glued) either to beams' soffit in flexural applications or vertically (perpendicular or inclined) on beams' web for shear strengthening applications. This procedure optimizes the use of FRP for flexural strengthening of RC beams (i.e. costs) and most of all ensures coherent and adequate transfer of forces between the RC beam and EB-FRP system [25]. While this procedure is proven effective in most loading scenarios, it may not be suitable or applicable in certain situations such as those associated with unique architectural arrangements, especially in beams with narrow/tapered geometric configurations such as double-tees, or in beams with limited working space/accessibility (attached to partition walls or spanning to multiple compartments) [25-27].

In order to overcome some of these challenges, bonding of FRP strengthening systems to the sides of beam's web in the longitudinal direction to enhance flexural capacity of RC beams has been identified as a possible solution [25-28]. This technique is referred to herein as side-bonding (SB) and is to be distinguished from that commonly used in shear strengthening applications, where FRP sheets are oriented vertically at 90° or inclined at 45° to resist the actions of resulting shear forces. In SB applications, FRP sheets or plates are glued along the bottom surface of the web, where the longitudinal fibers run parallel to the beam's soffit. As a result, FRP sheets act similar to tensile steel reinforcement and contribute to the bending and shear capacity of RC beams as well as control of crack development.

Only few researchers examined the effectiveness of externally side-boned (ESB) systems in RC beams [26-28]. In one study, Li et al. [26] compared the performance of a number of RC cantilever beams strengthened with carbon based FRP (CFRP) laminates. Two sets of beams were tested, one group was strengthened with traditional FRP systems (applied to the bottom of beams' soffit), and another set in which the RC beams were strengthened by SB technique. The main outcome of this study noted how both sets of RC beams achieved comparable performance and flexural stiffness. Li et al. [26] also reported that SB-FRP systems led to a lower crack width and reduced crack pattern in FRP-strengthened beams, which translated to a significant increase in the pre-crack stage of FRP-strengthened RC beams. In another study, Hosen et al. [27] experimentally investigated the flexural behavior of six RC beams strengthened with side-near-surface-mounted (SNSM) steel and CFRP bars, while being loaded under four-point bending. Overall, RC beams with SB-CFRP bars outperformed those with steel bars. More specifically, cracking load, yield and ultimate load-carrying capacities of the beams strengthened with CFRP rods increased by about 2.2, 1.0, and 1.4 times that of an unstrengthened RC beam, respectively. Similar to the tests carried out by Li et al. [26], the RC beams strengthened with CFRP bars experienced less cracking than that with steel bars. In a more recent work, the research group of the authors carried out an experimental study to explore external flexural strengthening of RC beams with side-bonded CFRP sheets [28-30]. This study indicated that CFRP side bonded RC beams showed a notable increase in flexural capacity. This gain in strength increases with the increase in CFRP magnitude, but seems to come with a significant drop in overall ductility.

A closer examination of the open literature shows that there is limited work on FRP-strengthening of RC beams through side-bonding technique. In fact, mechanisms of force transfer as well as critical parameters governing the effectiveness of SB systems are still not well understood, especially to practicing engineers and consultants. In order to bridge this knowledge gap, the present study aims to investigate critical parameters that influence flexural performance of RC beams externally strengthened with side-bonded FRP sheets through advanced numerical simulations. These simulations are carried out through a three dimensional (3D) finite element (FE) model developed in ANSYS software [31]. This model was specifically developed to accurately trace response of RC beams strengthened with side-bonded FRP systems by incorporating state-of-the-art simulation techniques including nonlinear material properties such as concrete cracking and crushing, yielding of embedded steel reinforcement and bond-slip action between FRP systems and concrete surfaces. The numerical results in terms of the load-carrying capacity and load versus mid-span deflection curves are compared with obtained experimental results. The developed and validated FE model was then expanded to investigate the effect of concrete compressive strength, FRP material type and size, and steel reinforcement

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ratio on flexural behavior of FRP-strengthened RC beams with side-bonded technique. The developed FE model could also serve as tool for designers and engineers seeking to utilize externally applied side-bonded FRP systems in retrofitting and strengthening applications.

2.0 Description of experimental program

The developed FE model is developed to investigate flexural behavior of recently tested RC beams strengthened with side bonded FRP sheets [28-30] by the research group of the authors. While full details on tested RC beams are spared herein for brevity, this section summaries the aforementioned experimental program and highlights main outcomes of those tests.

2.1 Geometric features

Nine RC beams were cast using normal aggregate concrete with a compressive strength of 47.2 MPa. These beams were made of rectangular cross section (300×150 mm) and spanned over 1700 mm. Those beams were reinforced with steel reinforcement composing of two 12 mm diameter bars in the tensile zone and two 8 mm diameter bars in the compression zone of the beam's cross-section. Those beams were also reinforced with 10 mm stirrups, spaced at intervals of 100 mm to ensure flexural failure of the control and strengthened beam specimens. All bars had a clear cover of 25 mm. Out of these nine beams, two beams were strengthened with SB-CFRP sheets, and hence are utilized in this study, together with a control unstrengthened RC beam specimen. A full description and designation of the tested specimen are provided in Table 1. The designation of these specimens for FE modeling is also provided in Table 1. One of the beam specimens was strengthened with 50 mm wide CFRP sheets, while the other was

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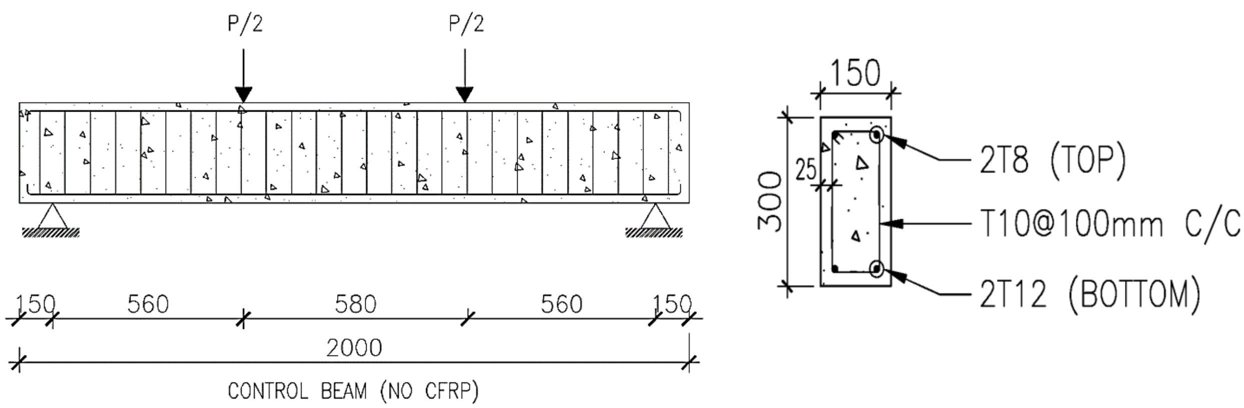
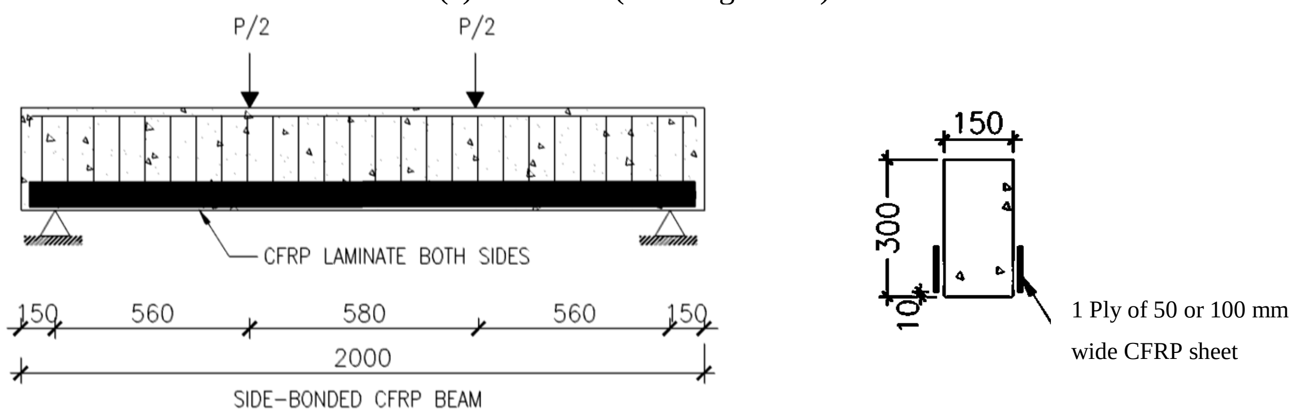
strengthened with 100 mm wide sheets. The CFRP sheets were bonded throughout the total length of the beam specimens along their longitudinal axis via epoxy adhesive. Figure 1 shows a general elevation view of the control and side-bonded strengthened RC beam specimen, as well as their cross-sectional details, respectively.



(a) RC beam (unstrengthened)



(b) RC beam (strengthened)

Fig. 1: Geometric details of tested and modeled RC beam specimens

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Table 1 Description of selected RC beams for analysis

Beam	**Designation as in tests [28-29]**	**Designation for developed FE models**	**FRP Type**	**FRP Width (mm)**
Control Beam (un-strengthened)	GB-C	CB03	N/A	N/A
Strengthened Beam with 100 mm wide CFRP sheets	GB-SS	SRB05	CFRP	100
Strengthened Beam with 50 mm wide CFRP sheets	GB-SS3	MB SS03	CFRP	50

2.2 Material properties

All RC beams were cast using C45 concrete, with a minimum compressive strength of 45 MPa. The obtained average cylindrical compressive strength (f'_c) after 28 days of casting was 47.2 MPa. The steel bars and stirrups that were used to reinforce the RC beam specimens are hot-rolled deformed BS4449 Grade 460 bars. Uniaxial coupon tensile tests were conducted to examine the mechanical properties of the utilized reinforcing bars. The obtained average elastic modulus, yield strength, and tensile strength were 199.9 GPa, 551.5 MPa, and 640.2 MPa, respectively.

Both side-bonded RC beams were strengthened with V-wrap C200H unidirectional CFRP sheets manufactured by Structural Technologies [32]. The dry sheets have mechanical properties reported at 227.5 GPa, 4830 MPa, and 2.1% for elastic modulus, tensile strength, and elongation at break, respectively. The average (obtained experimentally) and design values of the mechanical properties of cured composite CFRP sheets (laminates) impregnated with V-wrap 700 epoxy adhesives are provided in Table 2. The CFRP sheets were bonded via V-wrap 700

[33] epoxy adhesive. The elastic modulus, tensile strength, elongation at break, and glass transition temperature (T_g) of the epoxy adhesive are 3.18 GPa, 72.4 MPa, 5%, and 82 °C, respectively [33]:

Table 2: CFRP cured laminate properties [33]

Property	**Average Value**	**Design Value**
Tensile Strength (MPa)	1,240	1,034
Modulus of Elasticity (GPa)	73.77	73.77
Elongation at Break (%)	1.7	1.4
Thickness (mm)	1.02	1.02
Strength per Unit Width (kN/mm)	1.26	1.05

3.0 Finite Element model development

A finite element model has been developed to investigate the behavior of RC beams strengthened with side-bonded FRP laminates. ANSYS [31] software was used to create the FE models. The designation of the developed FE models that are compared with experimental results are provided in Table1. This section provides details of element types, material properties, loading and boundary conditions, and failure criteria of the developed FE models.

3.1 Concrete

An ANSYS Solid 65 concrete solid element was used to model the concrete beam specimens. This element can capture both tension cracks and compression crushing and defined by eight nodes with 3 translational degrees of freedom (*dof*) per node [31] along the x, y, and z directions, respectively. This element is also capable of simulating the nonlinear properties of concrete in compression and tension [31].

Three different concrete grades were used in the parametric study of this paper to investigate the effect of concrete compressive strength on the flexural behavior of the strengthened beam specimens. The material properties in terms of concrete compressive strength (f'_c), elastic modulus (E_c), Poisson ratio (ν), and tensile strength (f_t) for each concrete type are provided in Table 3. The concrete elastic modulus and tensile strength are computed as per the ACI318-14 [34] design guidelines.

Table 3. Concrete material properties

Concrete ID Type	f'_c (MPa)	E_c (GPa)	Poisson Ratio ν	f_t (MPa)
Type 1	25	23.5	0.2	3.10
Type 2	35	27.8	0.2	3.67
Type 3	47.2	32.2	0.2	4.25

The nonlinear properties of concrete under compression were assigned to the developed FE models throughout defining the strain-stress relation developed by Hognestad [35] as per Eqs. (1-3). The utilized stress-strain curve for each type of concrete is shown in Fig. 2.

$$f_c = f'_c\left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \qquad where \;\; 0 \le \varepsilon_c \le \varepsilon_{co} \qquad (1)$$

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$$f_c = f_c' - \left[\frac{0.15 f_c'}{\varepsilon_c - \varepsilon_{co}}\right](\varepsilon_c - \varepsilon_{co}) \quad where \quad \varepsilon_c > \varepsilon_{co} \tag{2}$$

$$\varepsilon_{co} = \frac{2 f_c'}{E_c} \tag{3}$$

where,

f_c = concrete compressive stress in MPa, corresponding to a specified strain value, ε_c

f'_c = concrete compressive strength in MPa

E_c = Modulus of concrete in MPa

ε_{co} = strain at peak compressive strength

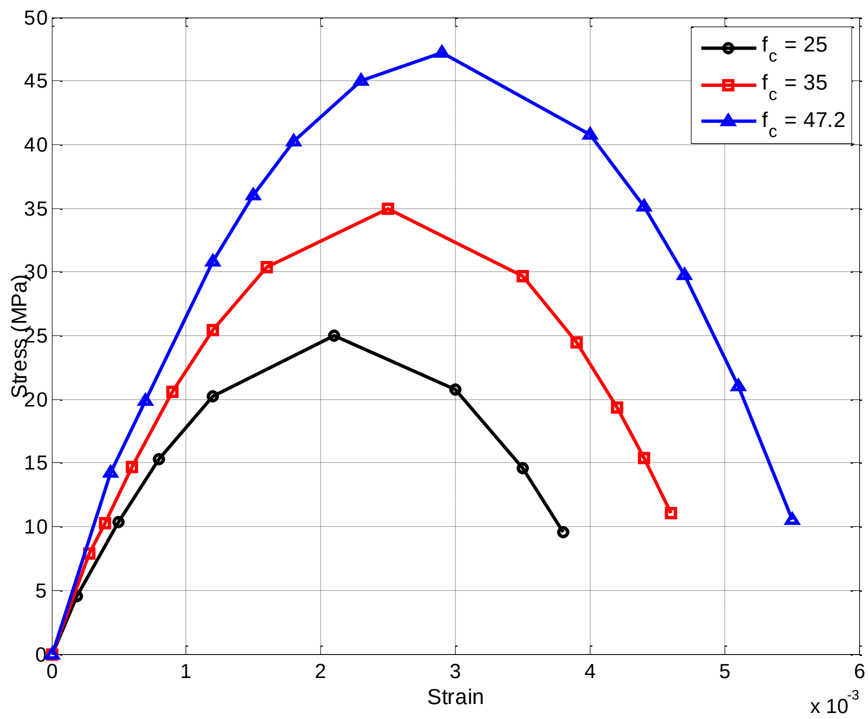

Fig. 2: Compression stress-strain curve for concrete

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The nonlinear material properties for concrete under tension employed in ANSYS is based on the William and Warnke model [36]. The tensile strength of concrete f_t is computed as per Eq. (4).

$$f_t = 0.62\sqrt{f_c'} \quad (4)$$

According to William and Warnke model, the concrete material in tension is modeled as linear elastic up to the concrete tensile (rupture) strength f_t. Once the tensile strength is reached, stress relaxation is simulated with a step drop of 40% of f_t and then followed by linear descending curve up to a strain value of $6\varepsilon_t$, where ε_t is the concrete strain value at f_t [31]. The concrete stress-strain curve in tension is shown in Fig. 3.

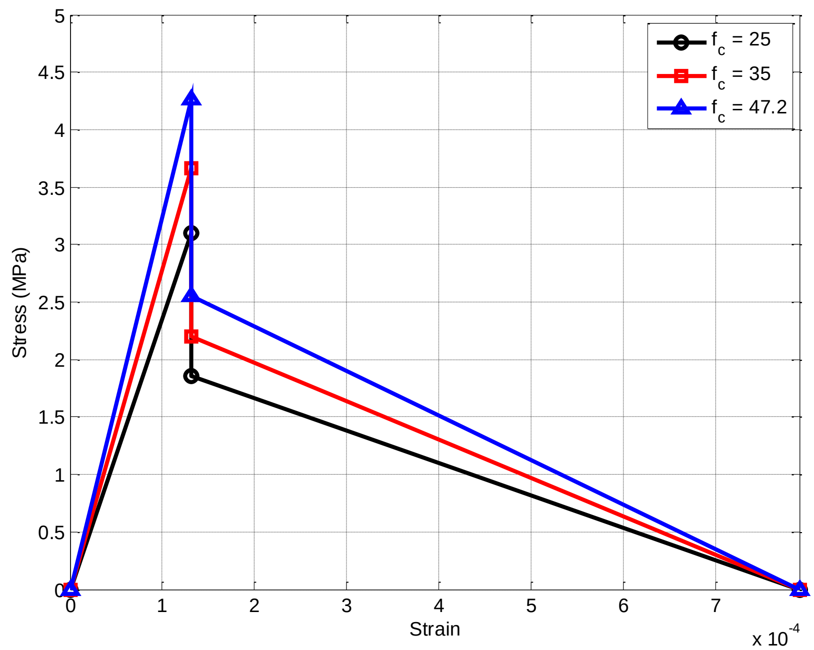

Fig. 3: Tensile stress-strain curve for concrete

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The concrete constitutive model in ANSYS [31] also requires values for the open and closed shear coefficients (β_t and β_c) that ranges from zero to one, where zero corresponds to smooth crack and no shear transfer and one corresponds to a rough crack with full shear transfer. Based on previous FE studies conducted by the authors, the values for β_t and β_c were taken as 0.3 and 0.5, respectively [17, 37-38].

3.2 Steel reinforcement

Steel reinforcement for both longitudinal bars and stirrups were modeled using ANSYS LINK180 elements [31]. This element is defined by two nodes and can take uniaxial tension and compression with the capability of elastic-plastic deformation, and has three DOF for each node.

The steel reinforcement was modeled as a nonlinear elastic-perfectly plastic material with an elastic modulus, yield strength, and Poisson ratio of 200 GPa, 550 MPa, and 0.3, respectively.

The above properties are in line with experimental test results reported in the experimental program of this study [28-30]. One of the variables in this numerical study is the flexural (bottom) steel reinforcement, with three different diameters of 12, 16, and 20 mm, respectively. In addition, 8 mm diameter bars were used for the top steel reinforcement and 10 mm diameter bars were used for the stirrups.

3.3 FRP laminates

An ANSYS SOLID185 element [31] was used to model the FRP laminates in the developed FE model. This element is similar to ANSYS SOLID 65 element, as it has eight nodes with 3 degrees of freedom for each node but without the capability of simulating concrete cracking and crushing. On the other hand, materials with orthotropic properties can be assigned to this element, where they can have different properties in each direction, which thus makes them suitable to model FRP laminates. All FRP sheets were modeled with a laminate thickness of 1.02 mm [29]. The orthotropic material properties of the different types of FRP laminates used in the developed FE models are summarized in Table 4, where:

E_x = Modulus of elasticity in the longitudinal x direction

E_y = Modulus of elasticity in the transverse y direction

E_z = Modulus of elasticity in the transverse z direction

v_{xy}= Poisson's ratio in the xy plane

v_{xz}= Poisson's ratio in the xz plane

v_{yz}= Poisson's ratio in the yz plane

G_{xy} = Shear modulus in the xy plane

G_{xz} = Shear modulus in the xz plane

G_{yz} = Shear modulus in the yz plane

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Table 4. Orthotropic mechanical properties of FRP laminates

FRP Type	E_x (MPa)	$E_y = E_z$ (MPa)	ν_{xz}	ν_{yz}	$G_{xy} = G_{xz}$ (MPa)	G_{yz} (MPa)
CFRP [29]	73,770	4,613	0.323	0.4	1,743	1,647
BFRP [39-40]	17,790	1,250	0.15	0.21	540	520
GFRP [41]	20,684	6,895	0.216	0.3	1,517	2,654

3.4 Load and support elements

The rigid steel loading supports were modeled using SOLID 185 [31] brick elements to eliminate any stress concentration on the concrete elements of the modeled RC beam specimens in the vicinity of the loading and reaction locations. The supports are also modeled with elastic material properties of 200 GPa for the elastic modulus and 0.3 for the Poisson ratio.

4.0 Finite Element models configuration and failure modes

In this paper, a total of fifteen FE models were developed to study the flexural behavior of RC beams externally bonded with side-bonded FRP laminates. Three specimens were used to validate the accuracy of numerical model by comparing with experimental results and the

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remaining models were developed to investigate the effect of concrete compressive strength, reinforcement ratio, and FRP sheets size and type on the flexural behavior of the beam.

To get advantage of symmetrical properties of the tested beam specimens in terms of geometry, loading, and boundary conditions, only one quarter of the specimen is built and analyzed in ANSYS [31] with two symmetrical planes along and across each beam specimens.

Figures 4 and 5 illustrate the developed FE models for two specimens; the first is control beam and the second model represents a RC beam externally strengthened with side-bonded FRP laminate.

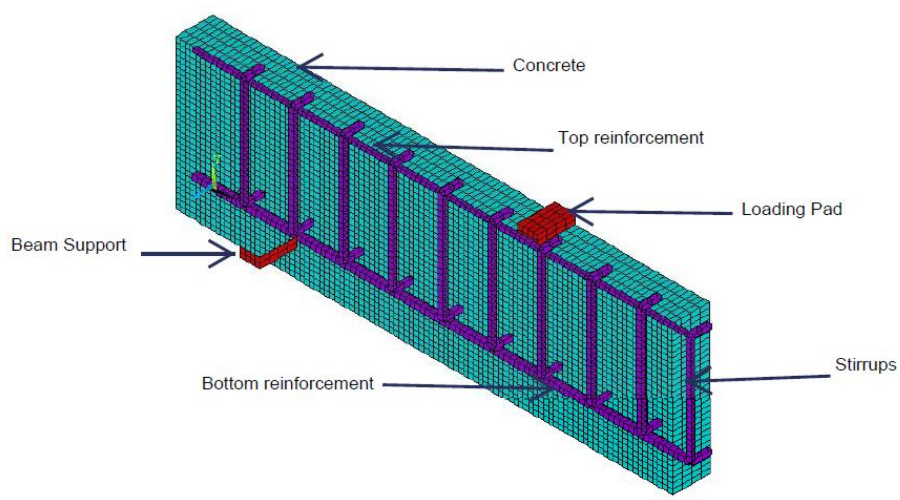

Fig. 4: Finite element model components for control beams

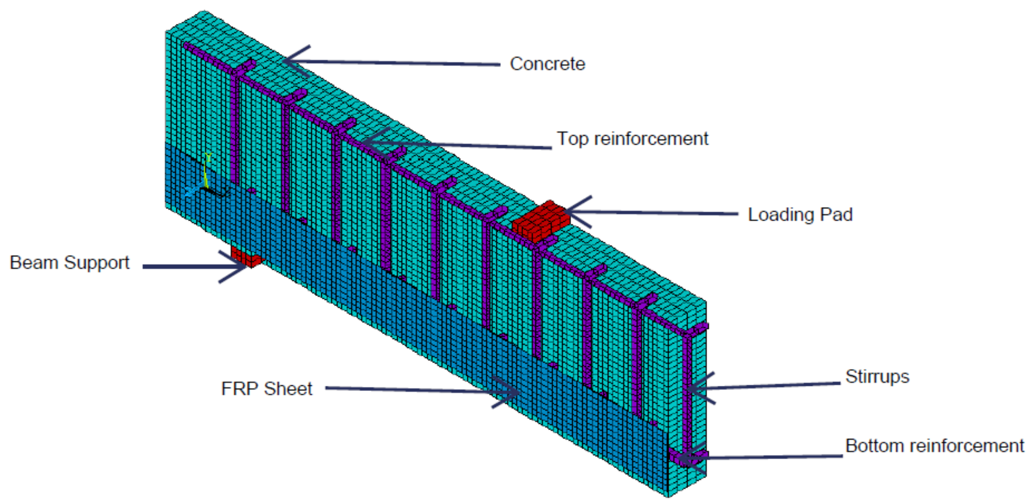

Fig. 5: Finite element model components for beams with FRP laminate

In order to anticipate the debonding of FRP laminate from adjacent concrete surface, a bond-slip stress in the FRP sheet has been computed based on the numerical model developed by Lu et al. [42] as presented in Eq. (5) – (9):

$$\tau = \begin{cases} \tau_{max}\sqrt{\frac{s}{s_0}} & \text{where } s \leq s_0 \\ \tau_{max} e^{-\alpha\left(\frac{s}{s_0}-1\right)} & \text{where } s > s_0 \end{cases} \tag{5}$$

$$\tau_{max} = 1.5\ \beta_w f_t \tag{6}$$

$$S_0 = 0.0195\ \beta_w^2 f_t \tag{7}$$

$$\alpha = \frac{1}{\frac{G_f}{\tau_{max} s_0} - \frac{2}{3}} \tag{8}$$

$$G_f = 0.308\ {\beta_w}^2 \sqrt{f_t} \tag{9}$$

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$$\beta_w = \sqrt{\frac{2.25 - \frac{b_f}{b_c}}{1.25 + \frac{b_f}{b_c}}}$$

where,

τ_{max}: maximum local bond stress in MPa

s : slip between concrete and FRP in mm

s_0 : local slip at τ_{max} in mm

β_w : width ratio of the FRP reinforcement to the concrete member

α : factor depends on interfacial fracture energy, bond strength,

and slip at the peak

G_f : interfacial fracture energy in MPa − mm

b_f : width of FRP sheet in mm

b_c : width of concrete section in mm

f_t : concrete tensile strength in MPa

Although this model accurately represents the bond-slip mechanism between concrete and FRP interfaces, it is used only as an indicator to predict the debonding failure mode, since Lu et al. [42] was generated for the case, when the FRP sheet is under pure tension when bonded to the beam's soffit. For side-bonded strengthening technique, the FRP laminates are subjected to tensile and shear forces, and thus, Lu et al. [42] is not applicable to predict the onset of debonding of FRP laminate from concrete surface. Further future studies are warranted to develop bond-slip analytical models for side-strengthended RC beam specimens. Failure is assumed to occur in the developed FE models if:

- Concrete strain at the top compression fibers reaches 0.003.
- Strain of FRP sheets reaches its ultimate rupture strain.
- Stress in the FRP laminate reaches its ultimate (maximum) local bond stress; i.e. when $\tau_{FRP} = \tau_{max}$. This is assumed to correspond to the FRP debonding failure mode.

It should be noted that the failure mode for the control beam specimens followed a typical behavior for an under-reinforced concrete beam, where the flexural steel bars yield first followed by concrete crushing between the two loading points. Regarding the RC beam specimens that were strengthened with side-bonded FRP laminates, they all failed by FRP debonding followed by concrete crushing.

5.0 FE Model validation

In order to validate and assure the accuracy of the numerical models, the three tested specimens that were explained in a preceding section were modeled and analyzed. The predicted and obtained experimental data are compared for a control unstrengthened beam specimen, and for two strengthened side-bonded specimens with CFRP sheets having widths of 100 mm and 50 mm respectively.

Figure 6 shows the experimental and predicted load versus mid-span deflection results at all stages of loading. In addition, Table 5 compares the predicted and experimentally measured ultimate attained load (P_u) along with its corresponding mid-span deflection (δ_u) values. In addition, Table 5 provides the ratio of the experimental over the predicted values for the ultimate loads ($P_{u,Exp}/P_{u,FE}$) and deflection ($\delta_{u,Exp}/\ \delta_{u,FE}$) values, respectively. It should be also noted that the response of the three specimens followed that of the tested specimens and failed in a similar mode.

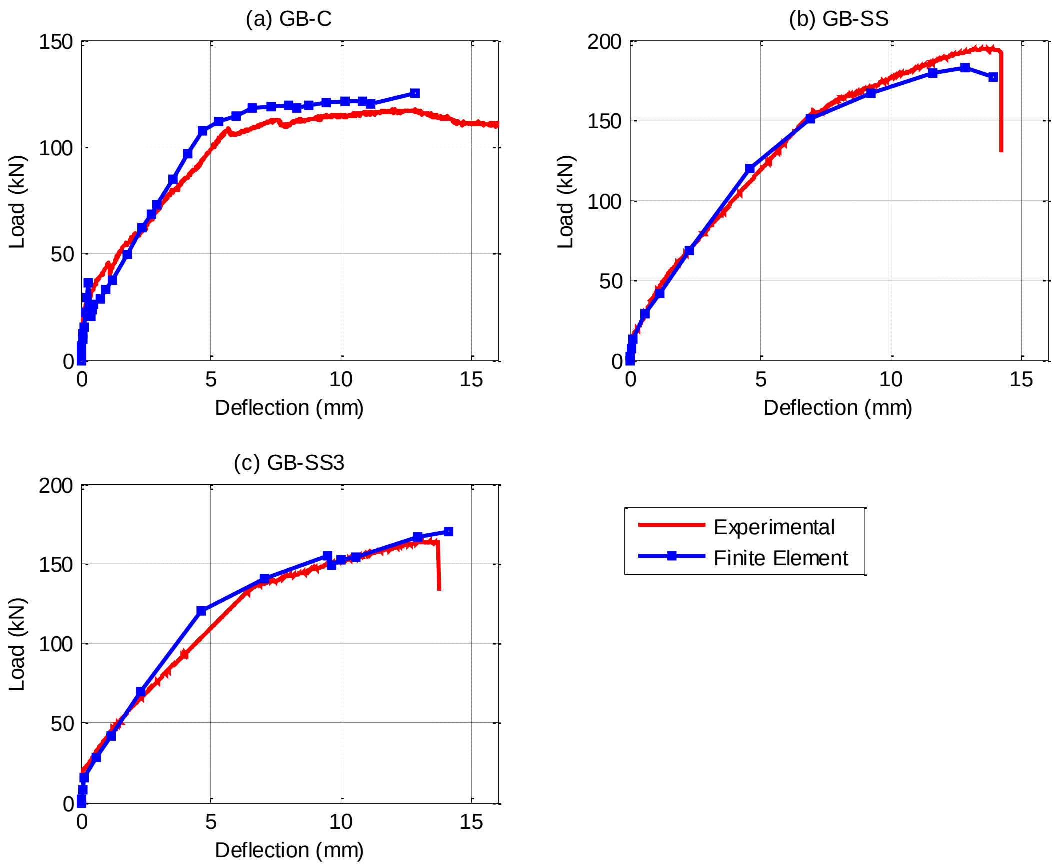

Fig. 1: Load versus mid-span deflection results for GB-C, GB-SS, and GB-SS3 specimens

Table 5. Results verification between experimental and FE developed models

Beam ID	P_u (kN)		$P_{u,Exp}$ / $P_{u,FE}$	δ_u (mm)		$\delta_{u,Exp}$/ $\delta_{u,FE}$
	Exp.	FE		Exp.	FE	
CB03	116.79	125.01	0.93	12.90	12.82	0.99
SRB05	194.56	190.49	1.02	13.70	12.66	1.082
MB SS03	163.59	167.00	0.98	13.40	12.91	1.038

It can be clearly seen from Fig. 6 and Table 5 that the predicted FE load and deflection results are in good agreement with the measured experimental data at all stages of loading. Furthermore, Table 5 shows that the difference in the results between the experimental and numerical results is less than 10%, which indicates that the developed FE models are valid, and could be used as a valid tool to predict the response and flexural strength of RC beams externally strengthened with side-bonded FRP laminates. The developed FE model could be also used in design oriented parametric studies to examine the effect of several parameters on the performance of RC beams externally strengthened with the side-bonded technique.

6.0 Parametric Study

In this section, a design oriented parametric study is performed by developing and analyzing twelve additional FE models to study the effect of the size (width) of CFRP laminates, concrete compressive strength, steel reinforcement ratio, and different types of FRP laminates on the flexural response and strength of RC beams externally strengthened with side-bonded laminates.

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6.1 Effect of CFRP sheet size

A total of four FE models were developed in order to investigate the beam's behavior when the size (width) of CFRP sheets is changed. One of the beams is modeled as a control unstrengthened specimen and the remaining three models were strengthened with side-bonded CFRP sheets, having different widths of 50, 100, and 150 mm, respectively. The analyzed models had a concrete compressive strength and steel reinforcement ratio of 35 MPa and 0.502%, respectively. The designation for each analyzed model in this sub-section is provided in Table 6.

The predicted load versus mid-span displacement response curves for CB02, SRB01, SRB02, and SRB03 beam models are shown in Fig. 7. In addition, the predicted ultimate attained load (P_u) along with its corresponding mid-span deflection (Δ_u), and failure mode are summarized and compared in Table 6.

Table 6. Effect of varying CFRP sheet size

Designation	CFRP width (mm)	P_u (kN)	% P_u increase over CB02	Δ_u (mm)	Failure Mode
CB02	-	120.36	-	12.42	Steel yielding followed by concrete crushing (flexural failure)
SRB01	50	174.66	45.1	16.35	FRP debonding followed by concrete crushing
SRB02	100	190.07	57.9	16.19	FRP debonding followed by concrete crushing
SRB03	150	202.33	68.0	16.22	FRP debonding followed by concrete crushing

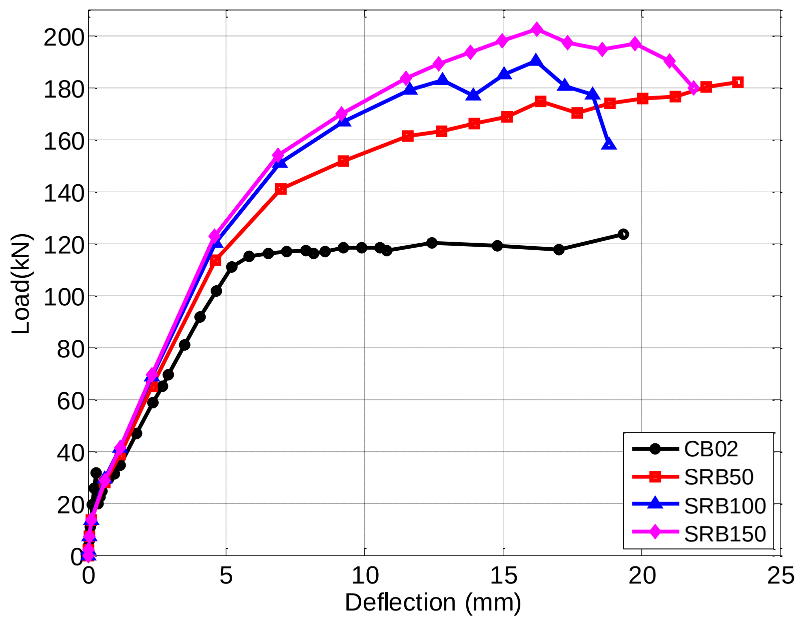

Fig. 7. Load versus deflection curves for specimens of different CFRP sizes

It can be clearly indicated from Fig. 7 and Table 6 that as expected, the beams with side bonded FRP sheets achieved a higher load-carrying capacity (flexural strength) as compared to the control beam (CB02). In particular, the load-carrying capacity (P_u) of beam SRB01 that has a 50 mm width CFRP laminate is higher than the control specimen (CB02) by 45.1%. It can be also noticed from Table 6 that the increase in P_u for beams SRB02 with 100 mm wide CFRP sheets and SRB03 with 150 mm wide CFRP sheets was 57.9% and 68.0% respectively. It could be thus concluded, that the percent increase in beam's load-carrying capacity is inversely proportional with the increase in the size (width) of CFRP laminates. This is due to the decrease in the depth of CFRP sheets, which will lead to a reduction in the moment arm of CFRP tensile force, and thus a reduction in the overall efficiency of side-bonded strengthening technique. Interestingly, it can be also noticed from Fig. 7 and Table 6 that the mid-span deflection (Δ_u) associated with P_u

for all beam specimens is comparable. Thus, the strength has been increased, while maintaining a reasonable level of ductility in the strengthened beam specimens. It could be also concluded that the side-strengthening technique would enhance both the strength and ductility of RC beams. This technique is somehow beneficial over the conventional soffit strengthening technique, where the strength of RC beams increases, however with a tremendous decrease in beam’s ductility.

6.2 Effect of concrete compressive strength

A total of six FE models were developed to study the effect of concrete compressive strength (f'_c) on the performance of RC beams externally strengthened with 100 mm wide side-bonded CFRP laminates. Three beams were modeled as control specimens and the other three models were strengthened with side-bonded CFRP laminates of different concrete compressive strength of 25, 35, and 47.2 MPa, respectively. The developed models had a steel reinforcement ratio of 0.502%. The beam designation for each developed FE model is provided in Table 7.

The predicted load versus mid-span displacement response curves for the developed models are provided in Fig. 8. In addition, the predicted ultimate attained load (P_u) along with its corresponding mid-span deflection (Δ_u), and failure mode are summarized and compared in Table 7.

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Table 7. Effect of varying concrete compressive strength

Designation	f'_c (MPa)	P_u (kN)	% P_u increase over control beams (CB)	Δ_u (mm)	Failure Mode
CB01	25	116.87	-	11.98	Steel yielding followed by concrete crushing
CB02	35	120.36	-	12.42	Steel yielding followed by concrete crushing
CB03	47.2	125.01	-	12.82	Steel yielding followed by concrete crushing
SRB04	25	156.35	33.8	9.05	FRP debonding
SRB02	35	190.07	57.9	16.19	FRP debonding
SRB05	47.2	190.49	52.4	12.66	FRP depонding

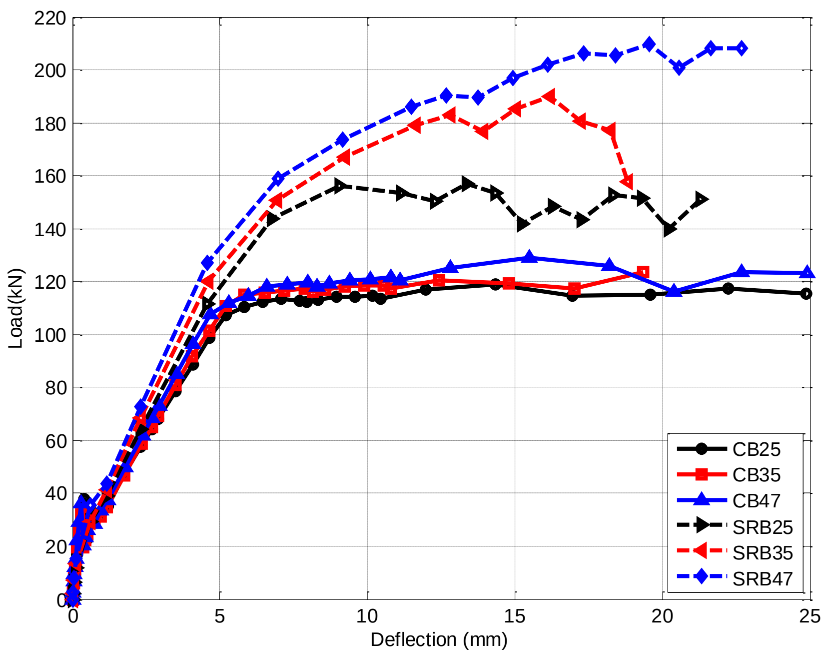

Fig. 8: Effect of concrete compressive strength on the performance of beams

It can be clearly indicated from Table 7 and Fig. 8 that the concrete compressive strength has a minimal effect on the performance of the control unstrengthened beam specimens. This is due to the fact that the flexural strength of under-reinforced concrete beams is dominated by the yield strength of steel reinforcement bars. However, the concrete compressive strength has a noticeable effect on the response and load-carrying capacity of RC beams strengthened with side-bonded CFRP laminates. In particular, the increment percent increase of the load-carrying capacity for specimens SRB04 (f'_c = *25 MPa*), SRB02 (f'_c = *35 MPa*), and SRB05 (f'_c = *47.2 MPa*) was 33.8, 57.9, and 52.4% more than their corresponding unstrengthened control beam specimens, respectively. This is due to the effect of the concrete tensile strength (f_t), which increases with the increase in f'_c, on the maximum local bond stress (τ_{max}) and bond-slip behavior between the CFRP laminates and the adjacent concrete surfaces, as indicated in Eqs. (5-9).

6.3 Effect of reinforcement ratio

A total of six FE models were developed to study the effect of steel reinforcement ratio (ρ) on the performance of RC beams externally strengthened with 100 mm wide side-bonded CFRP laminates. Three beams were modeled as control specimens and the remaining three were strengthened CFRP laminates with three different sizes of steel flexural bars (2#12, 2#16, and 2#20 mm bars) that corresponded to ρ values of 0.502, 0.889, and 1.395%, respectively. The utilized concrete compressive strength in the developed models was 35 MPa. The beam designation and a summary of test results for each developed FE model is provided in Table 8. In addition, Fig. 9 compares the predicted load versus mid-span deflection curves for the developed models.

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Table 8. Effect of varying steel reinforcement ratio

Designation	ρ (%)	P_u (kN)	% P_u increase over control beams (CB)	Δ_u (mm)	Failure Mode
CB02	0.502	120.36	-	12.42	Steel yielding followed by concrete crushing
CB04	0.889	190.92	-	9.60	Steel yielding followed by concrete crushing
CB05	1.395	226.31	-	6.63	Concrete crushing prior to steel yielding
SRB02	0.502	190.07	57.9	16.19	FRP debonding
SRB06	0.889	226.60	18.8	8.95	FRP debonding
SRB07	1.395	246.78	9	6.64	Concrete crushing prior to steel yielding

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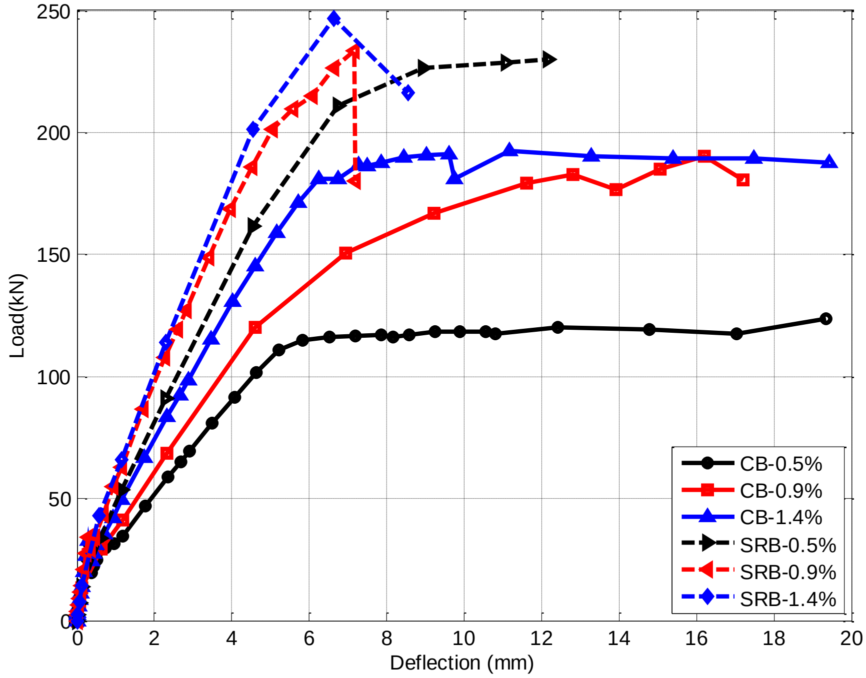

Fig. 9: Effect of steel reinforcement ratio on the performance of beams

It can be indicated from Table 8 and Fig. 9 that for the beams with low reinforcement ratio of 0.502%, the ultimate load-carrying capacity of strengthened beam has increased by 57.9% over the control beam, while for the beams with higher reinforcement ratio of 0.889 and 1.395%, the increase was only 18.8 and 9.0%, respectively over their corresponding control beams. It can be thus concluded that as the steel reinforcement ratio increases, from under reinforced to over reinforced, the percent increase in the flexural strength of side-bonded strengthened beam specimens decreases. Thus, side-bonded CFRP laminates are effective in strengthening RC beams with moderate amounts of flexural steel reinforcement ratio. However such CFRP strengthening system is not very effective in enhancing the flexural strength and performance of over-reinforced RC beams.

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6.4 Effect of FRP type

In order to study the effect of FRP type on the flexural behavior of RC beams with side-bonded FRP sheets, a total of four FE models were developed and analyzed. The first model was unstrengthened and the remaining three specimens were strengthened with 100 mm wide CFRP, BFRP and GFRP laminates, respectively. The analyzed models had a concrete compressive strength and steel reinforcement ratio of 35 MPa and 0.502%, respectively. The orthotropic material properties for those sheets are provided in Table 4. In addition, the beams' designation along with a summary of predicted results for P_u, Δ_u, and failure mode are presented in Fig. 10 and Table 9.

Table 9. Effect of varying FRP sheet type

Designation	FRP Type	P_u (kN)	% P_u increase over control beams (CB)	Δ_u (mm)	Failure Mode
CB02	-	120.36	-	12.42	Steel yielding followed by concrete crushing
SRB02	CFRP	190.07	57.9	16.19	FRP debonding
SRB08	BFRP	142.42	18.3	13.98	FRP debonding
SRB09	GFRP	148.58	23.4	16.50	FRP debonding

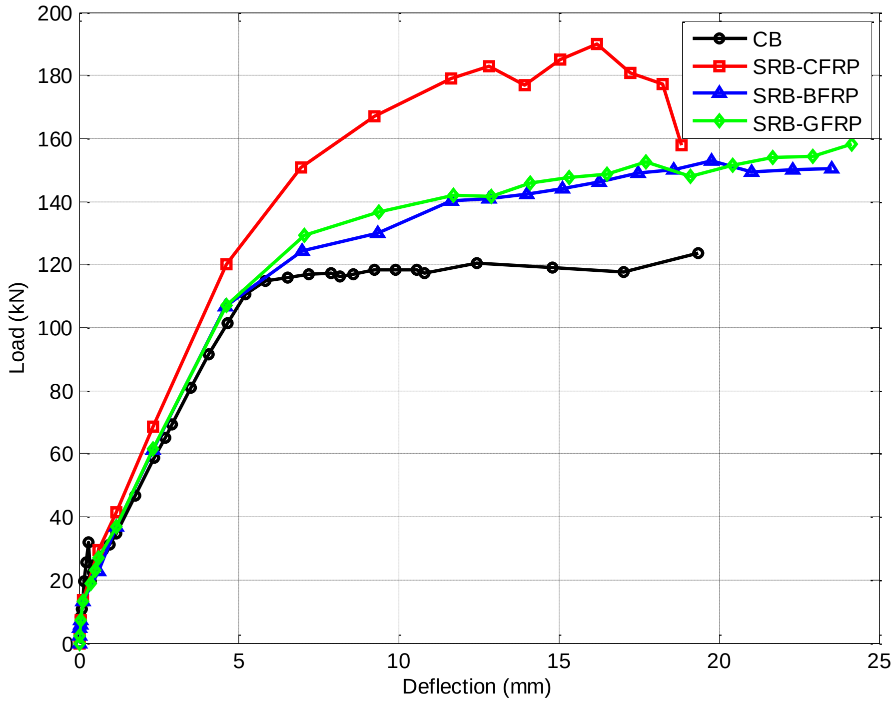

Fig. 10: Effect of FRP sheet type on the performance of beams

It can be indicated from Table 9 and Fig. 10 that the increase in the predicted load-carrying capacity of the RC beams strengthened with CFRP, BFRP, and GFRP specimens was 57.9, 18.3, and 23.4%, respectively, over the control unstrengthened specimen. In addition, it can be clearly seen from Fig. 10 that the BFRP and GFRP strengthened beams exhibited a similar response. Thus, it could be concluded that side-bonded CFRP laminates would provide the highest enhancement in flexural strength of RC beams. It should be noted that those predicted FE results are a function of FRP sheet properties that will vary if other manufacturer's material properties were assigned to the FRP laminates.

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7.0 Summary and Conclusion

This study presented the results of developed finite element models to numerically study the flexural behavior of reinforced concrete beams externally strengthened with side-bonded CFRP laminates. The models were verified by comparing the behavior and response of three beam specimens with their peers that were obtained from experimental tests. The results showed a good correlation between the numerical and experimental results at all stages of loading till failure. The developed and validated models were then utilized in a design oriented parametric study to investigate the effect of CFRP sheet size, concrete compressive strength, steel reinforcement ratio, and FRP type on the flexural performance of RC beams externally strengthened with side-bonded FRP laminates. The following observations and conclusions could be extrapolated from this study:

- The FE models were able to reasonably simulate the flexural behavior of the tested reinforced concrete beam specimens with and without side-bonded-CFRP laminates.
- An increase in the beam's stiffness and load-carrying capacity was observed for beams that were strengthened with side-bonded FRP sheets. However, this increment was accompanied with a decrease in the beam's ductility.
- The percent increase in beam's load-carrying capacity is inversely proportional with the increase in the size (width) of CFRP laminates. This is due to the decrease in CFRP sheet depth, which will lead to a smaller moment arm of the CFRP tensile force that will lead to a reduction in the overall efficiency of the side-bonded strengthening technique.
- The concrete compressive strength has a minimal effect on the performance of the strengthened specimens. In particular, the increase of the load-carrying capacity for

strengthened specimens with concrete compressive strength of 25, 35, and 47.2 MPa was 33.8, 57.9, and 52.4%, respectively.

- The steel reinforcement ratio has a significant effect on the performance of strengthened RC beams with side-bonded CFRP laminates. In particular the strength increase for beam specimens with reinforcement ratio of 0.502, 0.889, and 1.395% was 57.9, 18.8, and 9.0%, respectively.
- As the steel reinforcement ratio increases, the percent increase in the flexural strength of side-bonded strengthened beam specimens decreases. Thus, side-bonded CFRP laminates are effective in strengthening RC beams with moderate amounts of flexural steel reinforcement ratio.
- The FRP type and properties has a significant effect on the performance of strengthened RC beams. In particular, the beam that was strengthened with side bonded CFRP sheets has achieved the highest enhancement in strength of 57.9% over the control beam, while the other beams strengthened with BFRP and GFRP laminates exhibited an increase in flexural strength of 18.3 and 23.4%, respectively.

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